Week 14: Convolution and Filters

1 COMPUTING INFINITE AND CYCLIC CONVOLUTION

For the following pairs of vectors $\mathbf{f}$ and $\mathbf{g}$, compute
(1) the product $(\mathbf{f}\cdot\mathbf{g})$
(2) the (infinite) convolution $(\mathbf{f}*\mathbf{g})$
(3) the cyclic convolution $(\mathbf{f}\circledast\mathbf{g})$
(4) the Fourier transforms of $\mathbf{f}$, $\mathbf{g}$, and $(\mathbf{f}\circledast\mathbf{g})$

(A) $\mathbf{f}=[1,\,2,\,3,\,4]$ and $\mathbf{g}=[1,\,-1,\,1,\,-1]$

(B) $\mathbf{f}=[1,\,2,\,3,\,4]$ and $\mathbf{g}=[1,\,0,\,-1,\,0]$

(C) $\mathbf{f}=[1,\,2,\,3,\,4]$ and $\mathbf{g}=[3,\,1,\,-1,\,1]$

(D) $\mathbf{f}=[1,\,2,\,3,\,4]$ and $\mathbf{g}=[1,\,2,\,3,\,4]$

2 FILTERS

Find the vector $\mathbf{g}$ so that $\mathscr{F}\{\mathbf{f}\circledast\mathbf{g}\}$ results in the following (assume $\mathscr{F}\{\mathbf{f}\}=[c_0,\,c_1,\,c_2,\,c_3]$).

(A) $\mathscr{F}\{\mathbf{f}\circledast\mathbf{g}\}=[c_0,\,c_1,\,0,\,c_3]$

(B) $\mathscr{F}\{\mathbf{f}\circledast\mathbf{g}\}=[0,\,0,\,c_2,\,0]$

(C) $\mathscr{F}\{\mathbf{f}\circledast\mathbf{g}\}=[c_0,\,0,\,c_2,\,0]$

(D) $\mathscr{F}\{\mathbf{f}\circledast\mathbf{g}\}=[-c_0,\,c_1,\,-c_2,\,c_3]$

3 INVERSE FILTERS

Find the complex vector $\mathbf{d}$ so that the following hold (assume $\mathscr{F}\{[f_0,\,f_1,\,f_2,\,f_3]\}=\mathbf{c}$).

(A) $\mathscr{F}\{[f_0,\,f_1,\,0,\,0]\}=(\mathbf{c}\circledast\mathbf{d})$

(B) $\mathscr{F}\{[0,\,0,\,f_3,\,f_4]\}=(\mathbf{c}\circledast\mathbf{d})$

(C) $\mathscr{F}\{[f_0,\,0,\,f_2,\,0]\}=(\mathbf{c}\circledast\mathbf{d})$

(D) $\mathscr{F}\{[f_0,\,f_1,\,-f_2,\,-f_3]\}=(\mathbf{c}\circledast\mathbf{d})$

4 THEORY AND ADVANCED PROBLEMS

A) Read about Schönhage-Strassen multiplication in
`http://www.ams.org/samplings/feature-column/fcarc-multiplication`
Use this by hand to compute 2458×7421.